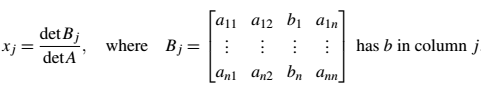
**determinant ( det*A* , |*A*| )**

* **properites of the determinant**(square matrices of any size)
  + 1. The determinant of the identity matrix is ***1***
    - det***I*** = *1*
  + 2. The determinant **changes sign** when two rows(columns) are exchanged
    - row exchange: \*(*-1*)
  + 3. The determinant **depends** linearly **on** **one single** row(column)
    - two operations
      * add vectors in one row(column)
      * multiply by t in one row(column)
  + 4. If two rows(columns) are equal, then the determinant is ***0***
    - det***A*** of equal rows = *0*
  + 5. Subtracting a multiple of one row from another row leaves **the same determinant**
    - elimination of rows(columns) doesn’t change the determinat
  + 6. If A has a row(column) of zeros, the determinant is ***0***
    - det***A*** of zero row = *0*
  + 7. If A is triangular, the determinant is **the product** of the diagonal entries
    - det***A*** of triangular matrix = the diagonal entries
  + 8. If A is singular, det***A*** = 0; If ***A*** is invertible, det***A*** != *0*
    - det***A***of singular = 0; det***A*** of nonsingular != *0*
  + 9. the det***AB***is **the product** of det***A*** times det***B***
    - |***AB***| = |***A***||***B***|
    - det***A-1*** = *1*/det***A*** (***A*** and ***B*** are nonsingular-->***AB*** is nonsingular)
  + 10. the **transpose** of ***A*** has **the same determinant** as ***A*** itself
    - |***A***| = |***AT***|
* **Big Formula**
  + 
    - detP = +1 or −1 for an even or odd number of row exchanges.
    - the number of permutation: ***n!***
* **det*A* by Cofactors**
  + det***A*** is a combination of any **row i** times **its cofactors**:
    - 
      * ***i*** belongs to **[*1,n*]**
  + The cofactor ***Cij*** is det***Mij*** with the correct sign:
    - 
  + since det***A*** = det***AT***
    - det***A*** can be a combination of any **column *i*** times **its cofactors**
      * 
* **application**
  + **compute *A-1***
    - ***A*** times **its transposed cofactor matrix** = **a diagonal matrix** with det***A****’s* on the diagonal
      * ***ACT*** = (det***A***)***I***
      * 
  + **solution of A*x* = *b***
    - **Cramer’s rule**: The *j* th component of *x* = ***A−1****b* is the ratio
    - 
      * since det***Bj*** = [*b1j b2j … bnj*][***C****1j,* ***C****2j, … ,* ***C****nj*] = *j* th component in the product ***CT****b*
      * for *n* equations Cramer's Rule would need *n+1* determinants
  + **the volume of a box**
    - det***A*** = volumn whose edges are the rows(columns) of ***A***
  + **a formula for the pivots**
    - Elimination can be completed without row exchanges, if and only if the leading **submatrices** ***A1, A2, … , An*** are all nonsingular
      * the **product** of the first *k* pivots is det ***Ak***
      * each pivot *dk* can be isolated as a ratio of determinants
        + formular for pivots : ***detAk / detAk-1 = dk***
    - **first *k* pivots** are determined by the submatrix ***Ak*** in upper left corner of ***A***
    - **upper-left corners** of ***L, D, U*** are determined by upper-left corner of ***A***